

New proposals for the detection of the Earth's gravitomagnetic field in space-based and laboratory-based experiments

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Abstract. In this contribution we present two new proposals for measuring the general relativistic gravitomagnetic component of the gravitational field of the Earth. One proposal consists of the measurement of the difference of the rates of the perigee ψ from the analysis of the laser-ranged data of two identical Earth-artificial satellites placed in equal orbits with supplementary inclinations. In this way the impact of the aliasing classical secular precessions due to the even zonal harmonics of the geopotential would be canceled out, although the non-gravitational perturbations, to which the perigees of LAGEOS-type satellites are particularly sensitive, should be a limiting factor in the obtainable accuracy. With a suitable choice of the inclinations of the orbital planes it would be possible to reduce the periods of such insidious perturbations so to use not too long observational time spans. However, the use of a pair of drag-free satellites would greatly reduce this problem, provided that the time span of the data analysis does not excess the lifetime of the drag-free apparatus. In the other proposal the difference of the rotational periods of two counter-revolving particles placed on a friction-free plane in a vacuum chamber at the South Pole should be measured in order to extract the relativistic gravitomagnetic signal. Among other very challenging practical implications, the Earth's angular velocity ω_{\oplus} should be known at a 10^{-15} rad s⁻¹ level from VLBI and the friction force of the plane should be less than 2×10^{-9} dyne.

1. The space based proposal

1.1. The Lense-Thirring effect

In the weak-field and slow-motion approximation of General Relativity, a test particle in the gravitational field of a slowly rotating body of mass M and angular momentum \mathbf{J} , assumed to be constant, is acted upon by a non-central acceleration of the form (Ciufolini and Wheeler, 1995; Ruggiero and Tartaglia, 2002)

$$\mathbf{a}_{\text{GM}} = \frac{\mathbf{v}}{c} \times \mathbf{B}_{\text{g}}, \quad (1)$$

in which \mathbf{v} is the velocity of the test particle, c is the speed of light in vacuum and \mathbf{B}_{g} is the gravitomagnetic field given by

$$\mathbf{B}_{\text{g}} = \frac{2G}{c} \frac{[\mathbf{J} - 3(\mathbf{J} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}}]}{r^3}. \quad (2)$$

In it $\hat{\mathbf{r}}$ is the unit position vector of the test particle and G is the Newtonian gravitational constant.

For a freely orbiting test particle eq. (1) induces on its orbit the so called Lense-Thirring drag of inertial frames (Ciufolini and Wheeler, 1995). More precisely, it consists of secular precessions of the

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longitude of the ascending node Ψ and of the argument of perigee ψ^2

$$\dot{\Psi}_{\text{LT}} = \frac{2GJ}{c^2 a^3 (1 - e^2)^{\frac{3}{2}}}, \quad (3)$$

$$\dot{\psi}_{\text{LT}} = -\frac{6GJ \cos i}{c^2 a^3 (1 - e^2)^{\frac{3}{2}}}, \quad (4)$$

in which i , a and e are the inclination, the semimajor axis and the eccentricity, respectively, of the orbit of the test body. Such general relativistic spin-orbit effect has been experimentally checked for the first time by analyzing the laser-ranged data to the LAGEOS and LAGEOS II artificial satellites in the gravitational field of the Earth with a claimed accuracy of the order of 20% – 30% (Ciufolini, 2002).

1.2. The LAGEOS-LARES mission

The use of the proposed LAGEOS III/LARES satellite would greatly increase the accuracy of such space-based measurement (Ciufolini, 1986). LARES would be a LAGEOS-type satellite to be placed in the same orbit as of LAGEOS except for the eccentricity, which should be one order of magnitude larger, and, especially, the inclination which should be supplementary to that of LAGEOS. Indeed, the proposed observable would be the sum of the nodal rates

$$\dot{\Psi}_{\text{LAGEOS}} + \dot{\Psi}_{\text{LARES}} \quad (5)$$

because, if on one hand, according to eq. (3), the Lense-Thirring nodal precessions are independent of the inclination and add up in eq. (5), on the other the aliasing classical nodal secular precessions induced by the even zonal harmonics of the Newtonian multipolar expansion of the terrestrial gravitational field (Kaula, 1966), which would represent a major source of systematic error, depend on $\cos i$ (Iorio, 2003d) and would be canceled out in eq. (5). However, as pointed out in (Iorio *et al*, 2002a), such a perfect cancellation would not occur due to the unavoidable orbital injection errors in the inclination of LARES. In (Iorio *et al*, 2002a) a revisited version of the LARES mission, including the orbital elements of LAGEOS and LAGEOS II as well, is presented. It would be more precise because it would cancel out the first four even zonal harmonics of the geopotential irrespectively of the departures of the LARES orbital parameters from their nominal values.

1.3. The difference of the perigees

The concept of satellites in identical orbits with supplementary orbital planes could be further exploited in order to obtain a new observable sensitive to the Lense-Thirring effect which is complementary to and independent of the already examined sum of the nodes. Indeed, from eq. (4) it turns out that the gravitomagnetic shift of the perigee depends on $\cos i$ while the classical secular precessions due to the even zonal coefficients of the geopotential depend on $\sin i$ and $\cos^2 i$ (Iorio, 2003d). Then, for a pair of satellites in supplementary identical orbits one could consider the difference of the perigees

$$\dot{\psi}^{(i)} - \dot{\psi}^{(180^\circ - i)} \quad (6)$$

because, while the Lense-Thirring precessions would be equal and opposite, the classical precessions due to the even zonal harmonics of the geopotential would be equal and would be canceled out in eq. (6) (Iorio, 2003a; 2003b; Iorio and Lucchesi, 2003).

With regard to the practical implementation of such an observable it should be noticed that the proposed LAGEOS-LARES mission would be unsuitable because the perigee is not a good observable for LAGEOS due to the extreme smallness of the eccentricity of its orbit ($e_{\text{LAGEOS}} = 0.0045$). Moreover, it should be noticed that, contrary to the nodes, the perigees of geodetic LAGEOS-like satellites are very sensitive to many time-dependent orbital perturbations of gravitational and, especially, non-gravitational origin whose periodicities are linear combinations of those of the lunisolar ecliptical

² The longitude of the ascending node Ψ is the angle, in the equatorial plane of an inertial frame whose origin is located at the center of mass of the central body, from a reference direction X , say the Aries point Υ , to the line of the nodes, i.e. the intersection of the orbital plane with the equatorial plane. The argument of perigee ψ is an angle in the orbital plane from the line of the nodes to the direction of the perigee.

variables and of the node and the perigee of the satellite itself. As showed in (Iorio, 2003a; Iorio and Lucchesi, 2003), many non-gravitational perturbations are not cancelled out by eq. (6); if the observational time span to be adopted for the data analysis is shorter than some of their periods it may happen that some uncanceled time-varying perturbations would corrupt the measurement of the investigated Lense-Thirring effect. The optimal choice would be the use of an entirely new pair of LAGEOS-type satellites, suitably built up in order to reduce the impact of the non-gravitational perturbations by reducing, e.g., the area-to-mass ratio, with identical eccentric orbits (say $e \sim 0.04$) in supplementary orbital planes with $i = 63.4^\circ$. With this choice of the inclination the periods of many uncanceled non-gravitational perturbations affecting eq. (6) would amount to only a few years, so that not too long observational time spans would be required in order to reduce the impact of such subtle perturbations. Indeed, on one hand, if the adopted time span is a multiple of their periods they average out and, on the other, they could be viewed as empirically fitted quantities to be removed from the temporal series, at least to a certain extent. In particular, it could be possible to adopt for the data analysis just the first years of life of the satellites: in this way many useful and simplifying assumptions on the interplay between the satellites' spin behavior and some related non-gravitational perturbations of thermal origin, like the solar Yarkovsky-Schach and the Earth infrared Yarkovsky-Rubincam thermal thrusts could be safely done.

1.4. Conclusions

From quantitative investigations (Iorio, 2003a; Iorio and Lucchesi, 2003) it turns out that such a new configuration would yield great benefits also to the accuracy of the sum of the nodes. Indeed, according to pessimistic evaluations, for a pair of LAGEOS-like SLR satellites with $a = 12000$ km, $e = 0.04$ and $i = 63.4^\circ$ and a suitably chosen time span the total systematic error in the sum of the nodes would amount to almost 0.1%, which is better than that could be obtained with the LAGEOS-LARES mission, while the total systematic error in the difference of the perigees would be of the order of 5% (without removing any time-dependent signals and by assuming LAGEOS-like satellites), mainly due to the non-gravitational perturbations. It should be recalled that, when in the near future the new, more accurate data for the Earth's gravitational field from the CHAMP and GRACE missions will be available, the role played by the non-gravitational perturbations in the total error budget will be dominant. Of course, the use of drag-free satellites would be able to greatly reduce their impact on the proposed measurement; a data analysis time span of just a few years, as it would be obtained with the chosen critical inclination, would be particularly well suited in view of the finite lifetime of the drag-free apparatus. Moreover, with the data of the orbital elements of such new satellites it would also be possible to follow the multiresidual linear combination approach sketched in (Iorio *et al*, 2002a) in conjunction with the data from LAGEOS and LAGEOS II.

2. A gravitomagnetic clock effect on Earth

2.1. The gravitomagnetic field of the Earth

In this section we intend to present a possible new Earth-based laboratory experiment (Iorio, 2003c) which exploits, in a certain sense, the concept of the gravitomagnetic clock effect (Iorio *et al*, 2002b) of two counter-orbiting test particles along identical circular orbits. For other proposed gravitomagnetic laboratory-based experiments see chapter 6 of (Ciufolini and Wheeler, 1995), the references in (Iorio, 2003c) and (Ruggiero and Tartaglia, 2002).

According to the gravitational analogue of the Larmor theorem (Mashhoon, 1993), we could obtain eq. (1) by considering an accelerated frame rotating with angular velocity

$$\boldsymbol{\Omega}_{LT} = \frac{\mathbf{B}_g}{2c}. \quad (7)$$

Indeed, in it an inertial Coriolis acceleration

$$\mathbf{a}_{Cor} = 2\mathbf{v} \times \boldsymbol{\Omega}_{LT} \quad (8)$$

is experienced by the proof mass. Then, in a laboratory frame attached to the rotating Earth we could consider, apart from the true Coriolis inertial force, also the relativistic term of eq. (8). Such gravitomagnetic feature could be measured, in principle, in the following way.

2.2. The experiment

Let us choose an horizontal plane at, say, South Pole: here the Earth's angular velocity vector $\boldsymbol{\omega}_\oplus$ and $\boldsymbol{\Omega}_{\text{LT}}$ are perpendicular to it and have opposite directions. Let us choose as unit vector for the z axis the unit vector $\hat{\boldsymbol{\Omega}}_{\text{LT}}$, so that

$$\boldsymbol{\Omega}_{\text{LT}} = \frac{2GJ}{c^2 R_p^3} \hat{\mathbf{z}}, \quad (9)$$

$$\boldsymbol{\omega}_\oplus = -\omega_\oplus \hat{\mathbf{z}}, \quad (10)$$

$$\mathbf{g} = -g \hat{\mathbf{z}}, \quad (11)$$

where \mathbf{J} is the proper angular momentum of the Earth, R_p is the Earth polar radius and \mathbf{g} is the gravitoelectric acceleration to which, at the poles, the centrifugal acceleration does not contribute. At a generic latitude the same reasoning holds provided that the component of $\boldsymbol{\Omega}_{\text{LT}}$ along the local vertical is considered. It is

$$\boldsymbol{\Omega}_{\text{LT}}^{(v)} = -\frac{2GJ \cos \theta}{c^2 R^3} \hat{\mathbf{r}} \quad (12)$$

where θ is the colatitude counted from the North Pole. A particle which moves with velocity \mathbf{v} in the previously considered polar horizontal plane is acted upon by the Coriolis inertial force induced by the noninertiality of the terrestrial reference frame and also by the gravitational force of eq. (8). Such forces have the same line of action and opposite directions: in an horizontal plane at South Pole the resultant acceleration is $2v\tilde{\Omega} \equiv 2v(\Omega_{\text{LT}} - \omega_\oplus)$ and it lies, orthogonally to \mathbf{v} , in the aforementioned plane. Let us consider an experimental apparatus consisting of a friction-free horizontal plane placed in a vacuum chamber. Upon such a desk a small tungsten mass m , tied to a sapphire fiber of length l , tension \mathbf{T} and fixed at the other extremity, is put in a circular uniform motion. Indeed, the forces which act on m are the tension of the wire, the Coriolis inertial force and the Lense–Thirring gravitational force which are all directed radially; the weight force $\mathbf{W} = m\mathbf{g}$ is balanced by the the normal reaction \mathbf{N} of the plane and there are neither the atmospheric drag nor the friction of the plane. Let us assume the counterclockwise rotation as positive direction of motion for m . At the equilibrium the equation of motion is

$$m\omega_+^2 l = T - 2m\omega_+ l \tilde{\Omega}, \quad (13)$$

where ω_+ is the angular velocity of the mass m when it rotates counterclockwise and l is the radius of the circle described by m . If the Earth did not rotate the angular velocity of the particle would be

$$\omega_0 = \sqrt{\frac{T}{ml}}. \quad (14)$$

The gravitomagnetic and the Coriolis forces slightly change such circular frequency. Since $\omega_0 \gg \tilde{\Omega}$, from eq. (13) it follows for both the counterclockwise and clockwise directions of rotation

$$\omega_\pm = \omega_0 \mp \tilde{\Omega}, \quad (15)$$

so that we could adopt as observable

$$\Delta\omega \equiv \omega_- - \omega_+ = 2\tilde{\Omega} \equiv 2(\Omega_{\text{LT}} - \omega_\oplus). \quad (16)$$

Of course, the physical properties of the sapphire fiber and of the tungsten mass should not change from a set of rotations in a direction to another set of rotations in the opposite direction, so to allow an exact cancellation of ω_0 in eq. (16).

2.3. Discussion

Since on the Earth's surface at the poles $\Omega_{\text{LT}} = 3.4 \times 10^{-14} \text{ rad s}^{-1}$, we must ask if the experimental sensitivity of the sketched apparatus allows to measure such so tiny effect. If we measure the frequency shift $\Delta\omega$ from the rotational periods of the mass m we have

$$\delta\Omega_{\text{LT}} = \frac{\delta(\Delta\omega)^{\text{exp}} + \delta\omega_\oplus}{2} \quad (17)$$

with

$$\delta(\Delta\omega)^{\text{exp}} = \delta\omega_-^{\text{exp}} - \delta\omega_+^{\text{exp}} = 2\pi \left[\left(\frac{\delta P_-}{P_-^2} \right)^{\text{exp}} - \left(\frac{\delta P_+}{P_+^2} \right)^{\text{exp}} \right]. \quad (18)$$

The Earth's angular velocity ω_{\oplus} is very well known in a kinematically, dynamically independent way from the Very Long Baseline Interferometry (VLBI) technique with an accuracy of the order of³ $\delta\omega_{\oplus} \sim 10^{-18} \text{ rad s}^{-1}$. In fact, ω_{\oplus} is not exactly uniform and experiences rather irregular changes which are monitored in terms of Length-Of-Day (LOD) by the Bureau Internationale des Poids et Mesures-Time Section (BIMP) on a continuous basis⁴. Such changes are of the order of $\Delta\omega_{\oplus} \sim 0.25$ milliarcseconds per year (mas yr^{-1}) $= 3.8 \times 10^{-17} \text{ rad s}^{-1}$, so that they are negligible. A possible source of error might come from our uncertainty in the position of the proposed polar set-up with respect to the Earth's crust, i.e. from the polar motion of the instantaneous axis of rotation of the Earth $\hat{\omega}_{\oplus}$ in terms of the small angles x and y . It turns out that this phenomenon has three components: a free oscillation with a (measured) period of 435 days (Chandler Wobble) and an amplitude of less than 1 arcsecond (asec), an annual oscillation forced by the seasonal displacement of the air and water masses of the order of 10^{-1} asec and an irregular drift of the order of some asec. There are also some diurnal and semi-diurnal tidally induced oscillations with an amplitude less than 1 mas. As a consequence, the position of the pole is unknown at a level of some meters. The small size of the apparatus should overcome such problem. Moreover, it can be easily seen that the impact of such offsets of $\hat{\omega}_{\oplus}$ on the Coriolis force is $2v\omega_{\oplus} \cos \delta$ with δ of the order of some asec or less, so that it is negligible.

In regard to the experimental measurement of the periods P_{\pm} , it should be possible to strongly constrain eq. (18) by choosing suitably the parameters of the apparatus so to increase the periods and/or by measuring them after many revolutions. However, the important point is that their difference only is important, and it should be possible to reduce such difference to the accuracy level required.

Of course, we are aware of the fact that many practical difficulties would make the proposed measurement very hard to be implemented. For example, it turns out that the the friction force of the plane should be less than 2×10^{-9} dyne. Moreover, in order to reach the quoted accuracy in measuring ω_{\oplus} with VLBI several years of continuous observation would be required.

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³ See on the WEB: <http://www.iers.org/iers/products/eop/long.html> and <http://hpiers.obspm.fr/eop-pc/>.

⁴ For these topics see on the WEB: <http://einstein.gge.unb.ca/tutorial/> and <http://hpiers.obspm.fr/eop-pc/>.